INTERMEDIATE

Algebra Within Reach

6e

Ron Larson

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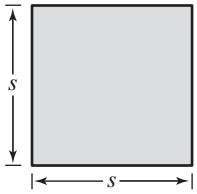
Geometry

Formulas for Area (A), Perimeter (P), Circumference (C), and Volume (V)

Square

$A = s^2$

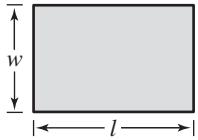
$P = 4s$



Rectangle

$A = lw$

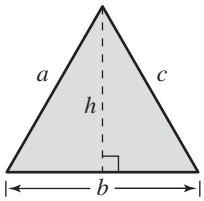
$P = 2l + 2w$



Triangle

$A = \frac{1}{2}bh$

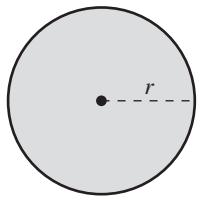
$P = a + b + c$



Circle

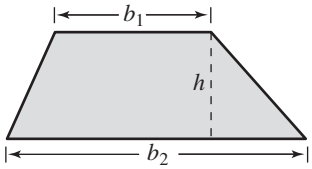
$A = \pi r^2$

$C = 2\pi r$



Trapezoid

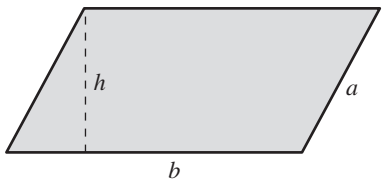
$A = \frac{1}{2}h(b_1 + b_2)$



Parallelogram

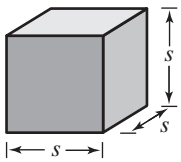
$A = bh$

$P = 2a + 2b$



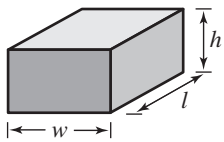
Cube

$V = s^3$



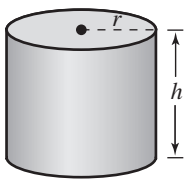
Rectangular Solid

$V = lwh$



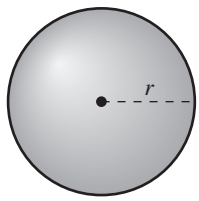
Circular Cylinder

$V = \pi r^2 h$



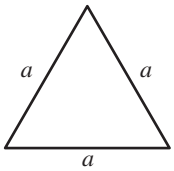
Sphere

$V = \frac{4}{3}\pi r^3$

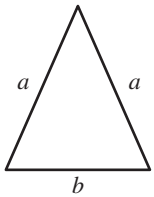


Special Triangles

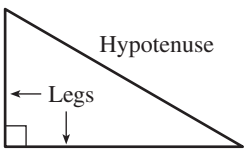
Equilateral Triangle



Isosceles Triangle

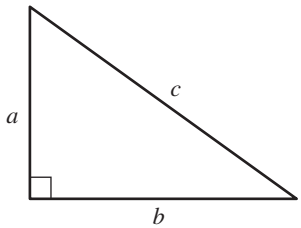


Right Triangle



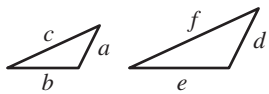
Pythagorean Theorem

$a^2 + b^2 = c^2$



Similar Triangles

$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$



Common Formulas

Distance

$d = rt$

d = distance traveled
t = time
r = rate

Temperature

$F = \frac{9}{5}C + 32$

F = degrees Fahrenheit
C = degrees Celsius

Simple Interest

$I = Prt$

I = interest
P = principal
r = annual interest rate
t = time in years

Compound Interest

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = balance
P = principal
r = annual interest rate
n = compoundings per year
t = time in years

Coordinate Plane: Midpoint Formula

Midpoint of line segment joining (x_1, y_1) and (x_2, y_2) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Coordinate Plane: Distance Formula

d = distance between points (x_1, y_1) and (x_2, y_2)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Quadratic Formula

Solutions of $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rules of Exponents

(Assume $a \neq 0$ and $b \neq 0$.)

$a^0 = 1$

$a^m \cdot a^n = a^{m+n}$

$(ab)^m = a^m \cdot b^m$

$(a^m)^n = a^{mn}$

$\frac{a^m}{a^n} = a^{m-n}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$a^{-n} = \frac{1}{a^n}$

$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$

Basic Rules of Algebra

Commutative Property of Addition

$a + b = b + a$

Commutative Property of Multiplication

$ab = ba$

Associative Property of Addition

$(a + b) + c = a + (b + c)$

Associative Property of Multiplication

$(ab)c = a(bc)$

Left Distributive Property

$a(b + c) = ab + ac$

Right Distributive Property

$(a + b)c = ac + bc$

Additive Identity Property

$a + 0 = 0 + a = a$

Multiplicative Identity Property

$a \cdot 1 = 1 \cdot a = a$

Additive Inverse Property

$a + (-a) = 0$

Multiplicative Inverse Property

$a \cdot \frac{1}{a} = 1, \quad a \neq 0$

Properties of Equality

Addition Property of Equality

If $a = b$, then $a + c = b + c$.

Multiplication Property of Equality

If $a = b$, then $ac = bc$

Cancellation Property of Addition

If $a + c = b + c$, then $a = b$.

Cancellation Property of Multiplication

If $ac = bc$, and $c \neq 0$, then $a = b$.

Zero Factor Property

If $ab = 0$, then $a = 0$ or $b = 0$.

INTERMEDIATE
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SIXTH EDITION

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Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of Kimberly Nolting
Hillsborough Community College

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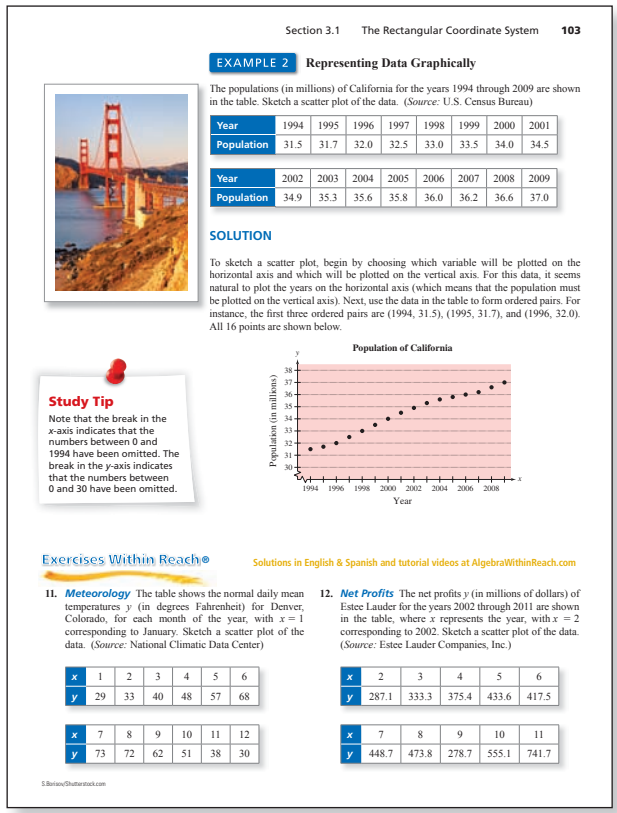
*Available at the text-specific website *www.cengagebrain.com*

Preface

Welcome to *Intermediate Algebra Within Reach*, Sixth Edition. I am proud to present to you this new edition. As with all editions, I have been able to incorporate many useful comments from you, our user. And, while much has changed with this revision, you will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook.

I'm very excited about this edition. As I was writing, I kept one thought in mind—provide students what they need to learn algebra *within reach*. As you study from this book, you should notice right away that something is different. I've structured the book so that examples and exercises are on the same page—*within reach*. I am also offering something brand new with this edition: a companion website at **AlgebraWithinReach.com**. This site offers many resources that will help you as you study algebra. All of these resources are just a click away—*within reach*.

My goal for every edition of this textbook is to provide students with the tools that they need to master algebra. I hope that you find the changes in this edition, together with AlgebraWithinReach.com, will accomplish just that.


Section 3.1 The Rectangular Coordinate System 103

EXAMPLE 2 **Representing Data Graphically**

The populations (in millions) of California for the years 1994 through 2009 are shown in the table. Sketch a scatter plot of the data. (*Source:* U.S. Census Bureau)

Year	1994	1995	1996	1997	1998	1999	2000	2001
Population	31.5	31.7	32.0	32.5	33.0	33.5	34.0	34.5

Year	2002	2003	2004	2005	2006	2007	2008	2009
Population	34.9	35.3	35.6	35.8	36.0	36.2	36.6	37.0

SOLUTION

To sketch a scatter plot, begin by choosing which variable will be plotted on the horizontal axis and which will be plotted on the vertical axis. For this data, it seems natural to plot the years on the horizontal axis (which means that the population must be plotted on the vertical axis). Next, use the data in the table to form ordered pairs. For instance, the first three ordered pairs are (1994, 31.5), (1995, 31.7), and (1996, 32.0). All 16 points are shown below.

Study Tip

Note that the break in the *x*-axis indicates that the numbers between 0 and 1994 have been omitted. The break in the *y*-axis indicates that the numbers between 0 and 30 have been omitted.

Population of California

Population (in millions)

Year

Exercises Within Reach® Solutions in English & Spanish and tutorial videos at AlgebraWithinReach.com

11. ***Meteorology*** The table shows the normal daily mean temperatures y (in degrees Fahrenheit) for Denver, Colorado, for each month of the year, with $x = 1$ corresponding to January. Sketch a scatter plot of the data. (*Source:* National Climatic Data Center)

x	1	2	3	4	5	6
y	29	33	40	48	57	68

x	7	8	9	10	11	12
y	73	72	62	51	38	30

12. ***Net Profits*** The net profits y (in millions of dollars) of Estee Lauder for the years 2002 through 2011 are shown in the table, where x represents the year, with $x = 2$ corresponding to 2002. Sketch a scatter plot of the data. (*Source:* Estee Lauder Companies, Inc.)

x	2	3	4	5	6
y	287.1	333.3	375.4	433.6	417.5

x	7	8	9	10	11
y	448.7	473.8	278.7	555.1	741.7

S.Borisov/Shutterstock.com

New To This Edition

REVISED Exercises Within Reach

The exercise sets have been carefully and extensively reviewed to ensure they are relevant and cover all topics suggested by our users. Additionally, the exercises have been completely restructured. Exercises now appear on the *same* page and immediately follow a corresponding example. There is no need to flip back and forth from example to exercise. The end-of-section exercises focus on mastery of conceptual understanding. View and listen to worked-out solutions at AlgebraWithinReach.com.

NEW Data Spreadsheets

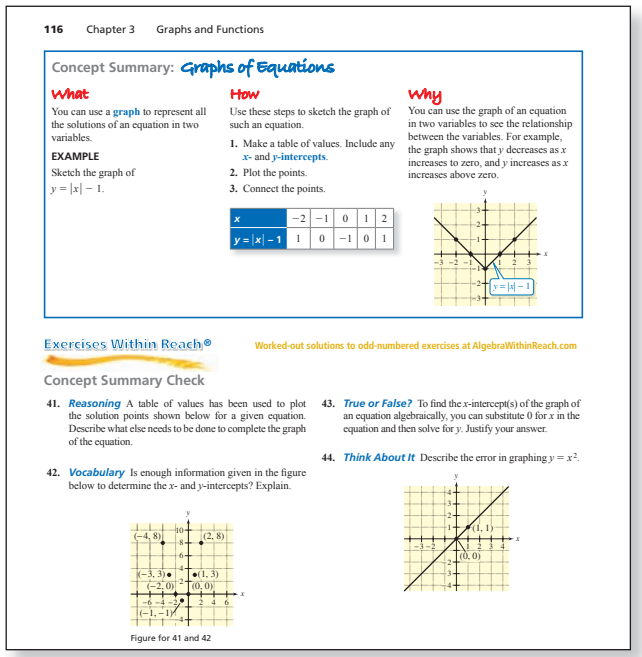
Download editable spreadsheets from AlgebraWithinReach.com, and use this data to solve exercises.

NEW AlgebraWithinReach.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. View and listen to worked-out solutions of thousands of exercises in English or Spanish, download data sets, take diagnostic tests, watch lesson videos and much more.

NEW Concept Summary

This simple review of important concepts appears at the end of every section. Each Concept Summary reviews *What*, *How*, and *Why*—what concepts you studied, how to apply the concepts, and why the concepts are important. The Concept Summary includes four exercises to check your understanding.


116 Chapter 3 Graphs and Functions

Concept Summary: Graphs of Equations

What

You can use a graph to represent all the solutions of an equation in two variables.

EXAMPLE

Sketch the graph of $y = |x| - 1$.

How

Use these steps to sketch the graph of such an equation.

1. Make a table of values. Include any x- and y-intercepts.
2. Plot the points.
3. Connect the points.

x	-2	-1	0	1	2
$y = \|x\| - 1$	1	0	-1	0	1

Why

You can use the graph of an equation in two variables to see the relationship between the variables. For example, the graph shows that y decreases as x increases to zero, and y increases as x increases above zero.

$y = |x| - 1$

Exercises Within Reach®

Worked-out solutions to odd-numbered exercises at AlgebraWithinReach.com

Concept Summary Check

41. **Reasoning** A table of values has been used to plot the solution points shown below for a given equation. Describe what else needs to be done to complete the graph of the equation.

42. **Vocabulary** Is enough information given in the figure below to determine the x- and y-intercepts? Explain.

(−4, 8) (2, 8) (−3, 3) (1, 3) (−2, 0) (0, 0) (−1, −1)

Figure for 41 and 42

43. **True or False?** To find the x-intercept(s) of the graph of an equation algebraically, you can substitute 0 for x in the equation and then solve for y. Justify your answer.

44. **Think About It** Describe the error in graphing $y = x^2$.

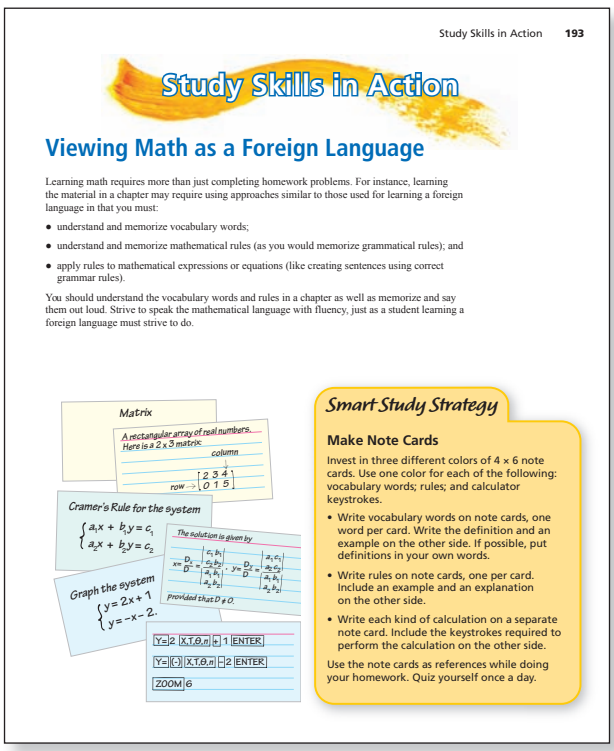
(1, 1) (0, 0)

NEW Math Helps

Additional instruction is available for every example and many exercises at AlgebraWithinReach.com. Just click on *Math Help*.

REVISED Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.


Study Skills in Action 193

Study Skills in Action

Viewing Math as a Foreign Language

Learning math requires more than just completing homework problems. For instance, learning the material in a chapter may require using approaches similar to those used for learning a foreign language in that you must:

- understand and memorize vocabulary words;
- understand and memorize mathematical rules (as you would memorize grammatical rules); and
- apply rules to mathematical expressions or equations (like creating sentences using correct grammar rules).

You should understand the vocabulary words and rules in a chapter as well as memorize and say them out loud. Strive to speak the mathematical language with fluency, just as a student learning a foreign language must strive to do.

Matrix

A rectangular array of real numbers. Here is a 2 × 3 matrix

column

row

Cramer's Rule for the system

The solution is given by

Graph the system

Smart Study Strategy

Make Note Cards

Invest in three different colors of 4 × 6 note cards. Use one color for each of the following: vocabulary words; rules; and calculator keystrokes.

- Write vocabulary words on note cards, one word per card. Write the definition and an example on the other side. If possible, put definitions in your own words.
- Write rules on note cards, one per card. Include an example and an explanation on the other side.
- Write each kind of calculation on a separate note card. Include the keystrokes required to perform the calculation on the other side.

Use the note cards as references while doing your homework. Quiz yourself once a day.

REVISED Study Skills in Action

Each chapter presents a study skill essential to success in mathematics. Read and apply these throughout the course. Print them out at AlgebraWithinReach.com to keep them as reminders to develop strong study skills.

REVISED Applications

A wide variety of real-life applications are integrated throughout the text in examples and exercises. These applications demonstrate the relevance of algebra in the real world. Many of these applications use current, real data.

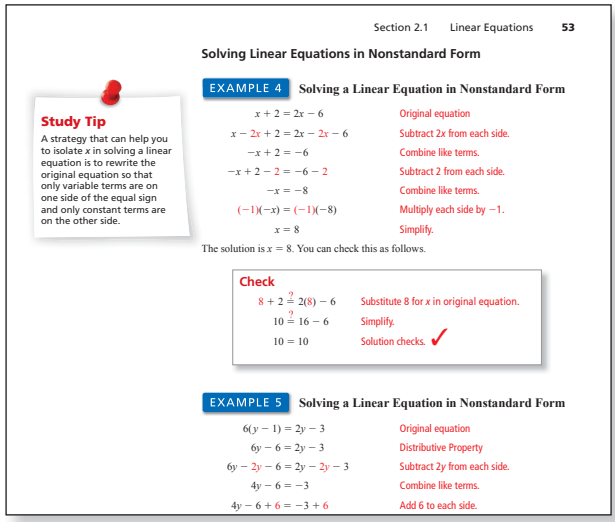
REVISED Chapter Summaries

The *Chapter Summary* now includes explanations and examples of the objectives taught in the chapter. Review exercises that cover these objectives are listed to check your understanding of the material.

Trusted Features

Examples

Each example has been carefully chosen to illustrate a particular mathematical concept or problem-solving technique. The examples cover a wide variety of problems and are titled for easy reference. Many examples include detailed, step-by-step solutions with side comments, which explain the key steps of the solution process.



Section 2.1 Linear Equations 53

Solving Linear Equations in Nonstandard Form

Study Tip
A strategy that can help you to isolate x in solving a linear equation is to rewrite the original equation so that only variable terms are on one side of the equal sign and only constant terms are on the other side.

EXAMPLE 4 Solving a Linear Equation in Nonstandard Form

$x + 2 = 2x - 6$	Original equation
$x - 2x + 2 = 2x - 2x - 6$	Subtract $2x$ from each side.
$-x + 2 = -6$	Combine like terms.
$-x + 2 - 2 = -6 - 2$	Subtract 2 from each side.
$-x = -8$	Combine like terms.
$(-1)(-x) = (-1)(-8)$	Multiply each side by -1.
$x = 8$	Simplify.

The solution is $x = 8$. You can check this as follows.

Check

$8 + 2 \stackrel{?}{=} 2(8) - 6$	Substitute 8 for x in original equation.
$10 \stackrel{?}{=} 16 - 6$	Simplify.
$10 = 10$	Solution checks. ✓

EXAMPLE 5 Solving a Linear Equation in Nonstandard Form

$6(y - 1) = 2y - 3$	Original equation
$6y - 6 = 2y - 3$	Distributive Property
$6y - 2y - 6 = 2y - 2y - 3$	Subtract $2y$ from each side.
$4y - 6 = -3$	Combine like terms.
$4y - 6 + 6 = -3 + 6$	Add 6 to each side.

Study Tips

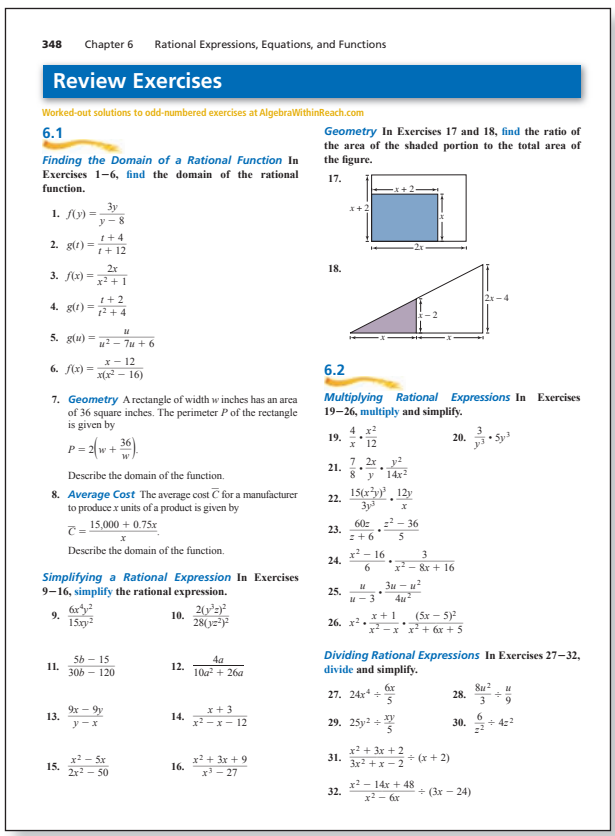
Study Tips offer students specific point-of-use suggestions for studying algebra, as well as pointing out common errors and discussing alternative solution methods. They appear in the margins.

Technology Tips

Point-of-use instructions for using graphing calculators or software appear in the margins as *Technology Tips*. These features encourage the use of graphing technology as a tool for visualization of mathematical concepts, for verification of other solution methods, and for help with computations.

Cumulative Review

Each exercise set (except in Chapter 1) is followed by *Cumulative Review* exercises that cover concepts from previous sections. This serves as a review and also a way to connect old concepts with new concepts.



348 Chapter 6 Rational Expressions, Equations, and Functions

Review Exercises

Worked-out solutions to odd-numbered exercises at AlgebraWithinReach.com

6.1

Finding the Domain of a Rational Function **In Exercises 1–6, find the domain of the rational function.**

1. $f(y) = \frac{3y}{y - 8}$
2. $g(t) = \frac{t + 4}{t + 12}$
3. $f(x) = \frac{2x}{x^2 + 1}$
4. $g(t) = \frac{t + 2}{t^2 + 4}$
5. $g(u) = \frac{u}{u^2 - 7u + 6}$
6. $f(x) = \frac{x - 12}{x(x^2 - 16)}$
7. ***Geometry*** A rectangle of width w inches has an area of 36 square inches. The perimeter P of the rectangle is given by $P = 2\left(w + \frac{36}{w}\right)$. Describe the domain of the function.
8. ***Average Cost*** The average cost $\overline{C}$ for a manufacturer to produce x units of a product is given by $\overline{C} = \frac{15{,}000 + 0.75x}{x}$. Describe the domain of the function.

Simplifying a Rational Expression **In Exercises 9–16, simplify the rational expression.**

9. $\frac{6x^4y^2}{15xy^2}$
10. $\frac{2(y^2z)^2}{28(yz^2)^2}$
11. $\frac{5b - 15}{30b - 120}$
12. $\frac{4a}{10a^2 + 26a}$
13. $\frac{9x - 9y}{y - x}$
14. $\frac{x + 3}{x^2 - x - 12}$
15. $\frac{x^2 - 5x}{2x^2 - 50}$
16. $\frac{x^2 + 3x + 9}{x^3 - 27}$

Geometry **In Exercises 17 and 18, find the ratio of the area of the shaded portion to the total area of the figure.**

17.

18.

6.2

Multiplying Rational Expressions **In Exercises 19–26, multiply and simplify.**

19. $\frac{4}{x} \cdot \frac{x^2}{12}$
20. $\frac{3}{y^3} \cdot 5y^3$
21. $\frac{7}{8} \cdot \frac{2x}{y} \cdot \frac{y^2}{14x^2}$
22. $\frac{15(x^2y)^3}{3y^3} \cdot \frac{12y}{x}$
23. $\frac{60z}{z + 6} \cdot \frac{z^2 - 36}{5}$
24. $\frac{x^2 - 16}{6} \cdot \frac{3}{x^2 - 8x + 16}$
25. $\frac{u}{u - 3} \cdot \frac{3u - u^2}{4u^2}$
26. $x^2 \cdot \frac{x + 1}{x^2 - x} \cdot \frac{(5x - 5)^2}{x^2 + 6x + 5}$

Dividing Rational Expressions **In Exercises 27–32, divide and simplify.**

27. $24x^4 \div \frac{6x}{5}$
28. $\frac{8u^2}{3} \div \frac{u}{9}$
29. $25y^2 \div \frac{xy}{5}$
30. $\frac{6}{z^2} \div 4z^2$
31. $\frac{x^2 + 3x + 2}{3x^2 + x - 2} \div (x + 2)$
32. $\frac{x^2 - 14x + 48}{x^2 - 6x} \div (3x - 24)$

Mid-Chapter Quiz

Each chapter contains a *Mid-Chapter Quiz*. View and listen to worked-out solutions at AlgebraWithinReach.com.

Chapter Review

The *Review Exercises* at the end of each chapter contain skill-building and application exercises that are first ordered by section, and then grouped according to the objectives stated at the start of the section. This organization allows you to easily identify the appropriate sections and concepts for study and review.

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Author: Ron Larson

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If you have suggestions for improving this text, please feel free to write to me. Over the past two decades I have received many useful comments from both instructors and students, and I value these comments very much.

Ron Larson
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1 Fundamentals of Algebra

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Cathy
Music

See page 27 for suggestions about keeping a positive attitude.

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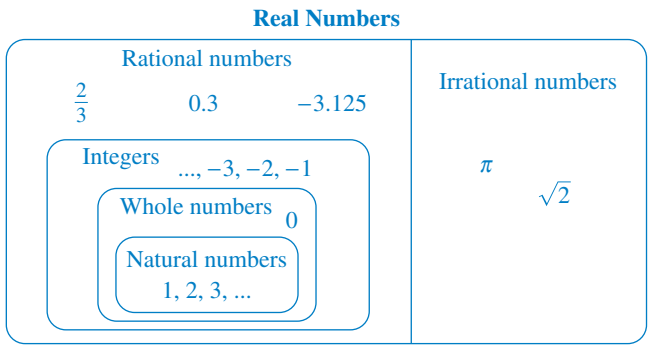
1.1 The Real Number System

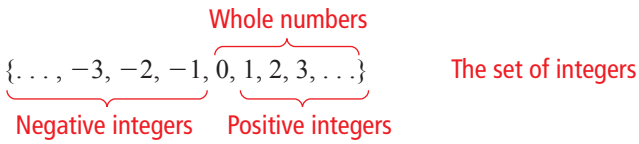
- Understand the set of real numbers and the subsets of real numbers.
- Use the real number line to order real numbers.
- Find the distance between two real numbers.
- Find the absolute value of a real number.

Real Numbers

The numbers you use in everyday life are called **real numbers**. They are classified into different categories, as shown at the right.



There are other classifications that are not shown in the diagram above. For instance, the set of **integers** can be divided into 3 categories: **negative integers**, zero, and **positive integers**.



EXAMPLE 1 Classifying Real Numbers

Which of the numbers in the set $\left\{-7, -\sqrt{3}, -1, -\frac{1}{5}, 0, \frac{3}{4}, \sqrt{2}, \pi, 5\right\}$ are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers?

SOLUTION

a. Natural numbers: $\{5\}$

b. Integers: $\{-7, -1, 0, 5\}$

c. Rational numbers: $\left\{-7, -1, -\frac{1}{5}, 0, \frac{3}{4}, 5\right\}$

d. Irrational numbers: $\left\{-\sqrt{3}, \sqrt{2}, \pi\right\}$

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Classifying Real Numbers **In Exercises 1–4, determine which of the numbers in the set are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers.**

1. $\left\{-6, -\sqrt{6}, -\frac{4}{3}, 0, \frac{5}{8}, 1, \sqrt{2}, 2, \pi, 6\right\}$

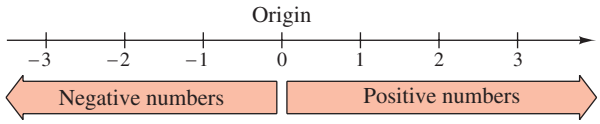
2. $\left\{-\frac{10}{3}, -\pi, -\sqrt{3}, -1, 0, \frac{2}{5}, \sqrt{3}, \frac{5}{2}, 5, 101\right\}$

3. $\left\{-4.2, \sqrt{4}, -\frac{1}{9}, 0, \frac{3}{11}, \sqrt{11}, 5.\overline{5}, 5.543\right\}$

4. $\left\{-\sqrt{25}, -\sqrt{6}, -0.\overline{1}, -\frac{5}{3}, 0, 0.85, 3, 110\right\}$

The Real Number Line and Order

The picture that represents the real numbers is called the **real number line**. It consists of a horizontal line with a point (the **origin**) labeled 0. Numbers to the left of zero are **negative** and numbers to the right of zero are **positive**.



The Real Number Line

Zero is neither positive nor negative. So, to describe a real number that might be either positive or zero, you can use the term **nonnegative real number**.

Each point on the real number line corresponds to exactly one real number, and each real number corresponds to exactly one point on the real number line. When you draw the point (on the real number line) that corresponds to a real number, you are **plotting** the real number.

EXAMPLE 2 Plotting Real Numbers

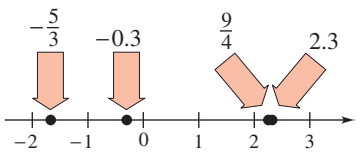
Plot each number on the real number line.

a. $-\frac{5}{3}$ **b.** 2.3

c. $\frac{9}{4}$ **d.** -0.3

SOLUTION

All four points are shown in the figure.



a. The point representing the real number $-\frac{5}{3} = -1.666\ldots$ lies between -2 and -1, but closer to -2, on the real number line.

b. The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

c. The point representing the real number $\frac{9}{4} = 2.25$ lies between 2 and 3, but closer to 2, on the real number line. Note that the point representing $\frac{9}{4}$ lies slightly to the left of the point representing 2.3.

d. The point representing the real number -0.3 lies between -1 and 0, but closer to 0, on the real number line.

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Plotting Real Numbers **In Exercises 5 and 6, plot the numbers on the real number line.**

5. (a) 3 (b) $\frac{5}{2}$ (c) $-\frac{7}{2}$ (d) -5.2

6. (a) 8 (b) $\frac{4}{3}$ (c) -6.75 (d) $-\frac{9}{2}$

Order on the Real Number Line

If the real number a lies to the left of the real number b on the real number line, then a is **less than** b, which is written as

$$a < b.$$

This relationship can also be described by saying that b is **greater than** a and writing $b > a$. The expression $a \leq b$ means that a is **less than or equal to** b, and the expression $b \geq a$ means that b is **greater than or equal to** a. The symbols $<$, $>$, $\leq$, and $\geq$ are called **inequality symbols**.

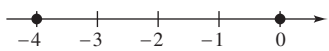
EXAMPLE 3 Ordering Real Numbers

Place the correct inequality symbol ($<$ or $>$) between the real numbers.

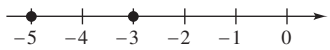
a. $-4 \quad 0$ **b.** $-3 \quad -5$ **c.** $\frac{1}{5} \quad \frac{1}{3}$ **d.** $-\frac{1}{4} \quad -\frac{1}{2}$

SOLUTION

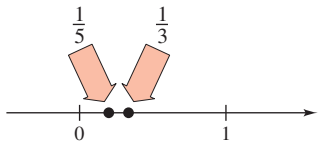
a. Because -4 lies to the left of 0 on the real number line, you can say that -4 is less than 0, and write $-4 < 0$.



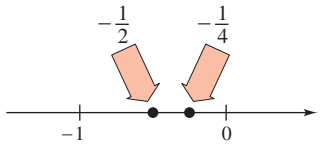
b. Because -3 lies to the right of -5 on the real number line, you can say that -3 is greater than -5, and write $-3 > -5$.



c. Because $\frac{1}{5}$ lies to the left of $\frac{1}{3}$ on the real number line, you can say that $\frac{1}{5}$ is less than $\frac{1}{3}$, and write $\frac{1}{5} < \frac{1}{3}$.



d. Because $-\frac{1}{4}$ lies to the right of $-\frac{1}{2}$ on the real number line, you can say that $-\frac{1}{4}$ is greater than $-\frac{1}{2}$ and write $-\frac{1}{4} > -\frac{1}{2}$.



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Ordering Real Numbers **In Exercises 7–16, place the correct inequality symbol ($<$ or $>$) between the real numbers.**

7. $\frac{4}{5} \quad 1$

8. $2 \quad \frac{5}{3}$

9. $-5 \quad 2$

10. $9 \quad -1$

11. $-5 \quad -2$

12. $-8 \quad -3$

13. $\frac{5}{8} \quad \frac{1}{2}$

14. $\frac{3}{2} \quad \frac{5}{2}$

15. $-\frac{2}{3} \quad -\frac{10}{3}$

16. $-\frac{5}{3} \quad -\frac{3}{2}$

Distance on the Real Number Line

Distance Between Two Real Numbers

If a and b are two real numbers such that $a \leq b$, then the **distance between a and b** is given by $b - a$.

Note from this definition that if $a = b$, the distance between a and b is zero. If $a \neq b$, then the distance between a and b is positive.

Study Tip

Recall that when you subtract a negative number, as in Example 4(a), you add the opposite of the second number to the first. Because the opposite of -2 is 2, you add 2 to 3.

EXAMPLE 4 Finding the Distance Between Two Numbers

Find the distance between the real numbers.

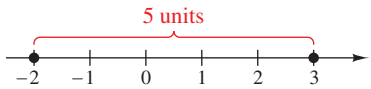
a. -2 and 3 **b.** 0 and 4

c. -4 and 0 **d.** 1 and $-\frac{1}{2}$

SOLUTION

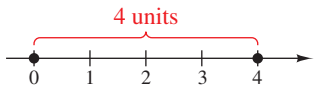
a. Because $-2 \leq 3$, the distance between -2 and 3 is

$$3 - (-2) = 3 + 2 = 5.$$



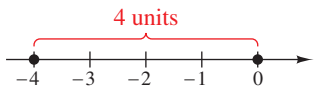
b. Because $0 \leq 4$, the distance between 0 and 4 is

$$4 - 0 = 4.$$



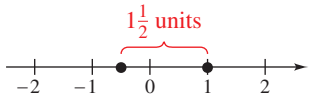
c. Because $-4 \leq 0$, the distance between -4 and 0 is

$$0 - (-4) = 0 + 4 = 4.$$



d. Because $-\frac{1}{2} \leq 1$, let $a = -\frac{1}{2}$ and $b = 1$. So, the distance between 1 and $-\frac{1}{2}$ is

$$1 - \left(-\frac{1}{2}\right) = 1 + \frac{1}{2} = 1\frac{1}{2}.$$



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Finding the Distance Between Two Numbers **In Exercises 17–28, find the distance between the real numbers.**

17. 4 and 10

18. 75 and 20

19. -12 and 7

20. -54 and 32

21. 18 and -32

22. 14 and -6

23. -8 and 0

24. 0 and 125

25. 0 and 35

26. -35 and 0

27. -6 and -9

28. -12 and -7

Finding Absolute Value

Two real numbers are called **opposites** of each other if they lie the same distance from, but on opposite sides of, 0 on the real number line.

Study Tip

Because *opposite* numbers lie the same distance from 0 on the real number line, they have the same absolute value. So, $|5| = 5$ and $|-5| = 5$.

Opposites and Additive Inverses

Let a be a real number.

1. $-a$ is the opposite of a.
2. $-(-a) = a$ Double negative
3. $a + (-a) = 0$ Additive inverse

Definition of Absolute Value

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}.$$

EXAMPLE 5 **Evaluating Absolute Values**

a. $|-10| = 10$ The absolute value of –10 is 10.

b. $\left|\frac{3}{4}\right| = \frac{3}{4}$ The absolute value of $\frac{3}{4}$ is $\frac{3}{4}$.

c. $-|-6| = -(6) = -6$ The opposite of $|-6|$ is -6.

EXAMPLE 6 **Comparing Real Numbers**

Place the correct symbol ($<$, $>$, or $=$) between the real numbers.

a. $|-2| \square 1$ **b.** $|-4| \square |4|$ **c.** $2 \square -|-2|$

SOLUTION

a. $|-2| > 1$, because $|-2| = 2$ and 2 is greater than 1.

b. $|-4| = |4|$, because $|-4| = 4$ and $|4| = 4$.

c. $2 > -|-2|$, because $-|-2| = -2$ and 2 is greater than -2.

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Evaluating an Absolute Value **In Exercises 29–34, evaluate the expression.**

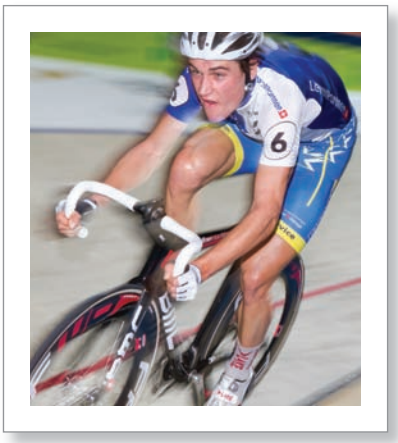
29. $|10|$ **30.** $|62|$ **31.** $|-225|$

32. $|-14|$ **33.** $-\left|-\frac{3}{4}\right|$ **34.** $-\left|\frac{3}{8}\right|$

Comparing Real Numbers **In Exercises 35–38, place the correct symbol ($<$, $>$, or $=$) between the real numbers.**

35. $|-6| \square |2|$ **36.** $|-2| \square |2|$ **37.** $|47| \square |-27|$ **38.** $|150| \square |-310|$

Application **EXAMPLE 7** **Translating Words into Symbols**



Write each statement using inequality symbols.

a. A bicycle racer's speed s is at least 16 miles per hour and at most 28 miles per hour.

b. The tire pressure p is at least 30 pounds per square inch and no more than 35 pounds per square inch.

c. The price p is less than \$225.

d. The average a will exceed 5000.

SOLUTION

a. $16 \le s \le 28$ (at least 16; at most 28)

b. $30 \le p \le 35$ (at least 30; no more than 35)

c. $p < 225$ (less than 225)

d. $a > 5000$ (will exceed 5000)

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Translating Words into Symbols **In Exercises 39–44, write the statement using inequality symbols.**

39. The weight on the elevator cannot exceed 2500 pounds.

40. You can drive at most 65 miles per hour on the interstate.

41. The contestant's weight must be more than 200 pounds.

42. You can save no more than \$2 with the coupon.

43. A person must be 52 inches tall or taller to ride the roller coaster.

44. It takes your friend at least 8 minutes and at most 10 minutes to run a mile.

45. ***Checking Account*** During the past month, the balance of your checking account did not exceed \$700 and did not drop below \$200. Write this statement using inequality symbols.

46. ***Reading*** Last night you read more than 40 pages and less than 70 pages of a new book. Write this statement using inequality symbols.

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Concept Summary: Ordering Real Numbers

What

When you are asked to order two **real numbers**, the goal is to determine which of the two numbers is greater.

EXAMPLE

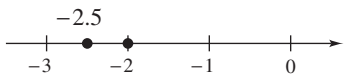
Order $-\frac{6}{3}$ and $-\frac{5}{2}$.

How

You can use the **real number line** to order two real numbers. For example, to order two fractions, rewrite them with the same denominator, or rewrite them as decimals. Then **plot** each number on a number line.

EXAMPLE

$$-\frac{6}{3} = -2, \quad -\frac{5}{2} = -2.5$$



$$-\frac{6}{3} > -\frac{5}{2}$$

Why

There are many situations in which you need to order real numbers. For instance, to determine the standings at a golf tournament, you order the scores of the golfers.

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Concept Summary Check

47. ***Using a Number Line*** Explain how the number line above shows that $-2.5 < -2$.

48. ***Using a Number Line*** Two real numbers are plotted on the real number line. How can you tell which number is greater?

49. ***Ordering Methods*** Which method for ordering fractions is shown in the solution above?

50. ***Rewriting Fractions*** Describe another way to rewrite and order $-\frac{6}{3}$ and $-\frac{5}{2}$.

Extra Practice

Identifying Numbers **In Exercises 51–54, list all members of the set.**

51. The integers between -5.8 and 3.2

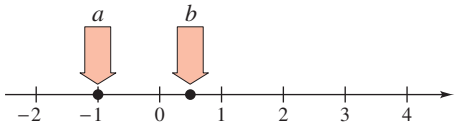
52. The even integers between -2.1 and 10.5

53. The odd integers between π and 10

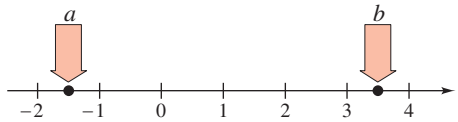
54. The prime numbers between 4 and 25

Approximating and Ordering Numbers **In Exercises 55–58, approximate the two numbers and order them.**

55.



56.



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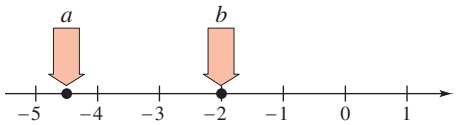

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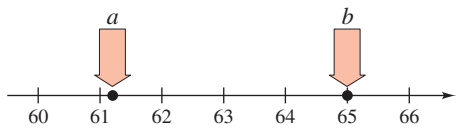

Evaluating an Absolute Value **In Exercises 59–64, evaluate the expression.**

59. $-|-85|$

60. $-|-36.5|$

61. $-|3.5|$

62. $|-1.4|$

63. $|-\pi|$

64. $-|\pi|$

Plotting Numbers **In Exercise 65–74, plot the number and its opposite on the real number line. Determine the distance of each from 0.**

65. -7

66. -4

67. 5

68. 6

69. $-\frac{3}{5}$

70. $\frac{7}{4}$

71. $\frac{5}{3}$

72. $-\frac{3}{4}$

73. -4.25

74. 3.5

Translating Words into Symbols **In Exercise 75–78, write the statement using inequality notation.**

75. x is negative.

76. y is more than 25.

77. u is at least 16.

78. x is nonnegative.

79. ***Coin Collection*** Write a statement represented by $30 < x < 50$, where x is the number of coins in a jar.

80. ***Basketball*** Write a statement represented by $280 \leq x \leq 310$, where x is the number points a basketball player scored this season.

Think About It **In Exercise 81–84, find two possible values of a.**

81. $|a| = 4$

82. $-|a| = -7$

83. The distance between a and 3 is 5.

84. The distance between a and -1 is 6.

Identifying Real Numbers **In Exercise 85–92, give three examples of numbers that satisfy the given conditions.**

85. A real number that is a negative integer

86. A real number that is a whole number

87. A real number that is a not a rational number

88. A real number that is not an irrational number

89. A rational number that is not an integer

90. A rational number that is not a negative number

91. A real number that is not a positive rational number

92. An integer that is not a whole number

Explaining Concepts

True or False? **In Exercises 93 and 94, decide whether the statement is true or false. Justify your answer.**

93. Every real number is either rational or irrational.

94. The distance between a number b and its opposite is equal to the distance between 0 and twice the number b.

95. ***Number Sense*** Compare the rational numbers 0.15 and $0.\overline{15}$.

96. ***Precision*** Is there a difference between saying that a real number is positive and saying that a real number is nonnegative? Explain your answer.